TRINITY COLLEGE

Mathematics Applications Unit 3/4
Test 5 2021

Calculator Assumed

Finance

STUDENT'S NAME SOLUTIONS

DATE: Thursday 12th August **TIME:** 50 minutes **MARKS**: 55

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (8 marks)

(a) If the simple interest on a sum of money at 5% per annum for 3 years is \$1200, what would the compound interest on the same sum for the same period at the same rate be? [3]

SI

$1200 = P \times 0.05 \times 3$ ✓

$\therefore P = \$8000$ ✓

CI

$I = 8000 \times 1.05^3 - 8000$ ✓

$= \$1261$ ✓ (3)

(b) (i) Calculate the value of a \$7100 investment in an account with a 3.20% per annum interest rate compounded daily after a period of 3 years. [1]

$A = 7100\left(1 + \frac{0.032}{365}\right)^{365 \times 3}$

$= \$7815.36$ ✓ (1)

(ii) Calculate the effective interest rate of this account. [1]

$\left(1 + \frac{0.032}{365}\right)^{365} - 1 = 3.25\%$ ✓ (1)

(iii) Under what circumstance would the 3.20% per annum interest rate have given the same effective interest rate. [1]

If it had been COMPOUNDED ANNUALLY ✓ (1)

(c) An investment produces \$205.57 after compounding interest quarterly for five years in a 1.8% per annum account. What amount of money was initially invested in the account? [2]

$P + 205.57 = P\left(1 + \frac{0.018}{4}\right)^{20}$ ✓

$\therefore P = \$2188$ ✓ (2)

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2. (9 marks)

Charles sets up a pension fund with $400 000 in an annuity with a growth rate of 5% compounded per annum. He plans to start withdrawing a payment of $45 000 each year starting January 1, 2022.

(a) For how many years will Charles be able to receive his annuity of $45 000? [3]

N ?
I 5
PV -400 000
PMT 45 000 ✓✓
FV 0
P/Y = C/Y = 1

N = 12 yrs ✓ (3)

(b) Charles wants to leave the $400 000 to his son William when he dies but still wants to take out regular amounts for himself each year. How can Charles ensure that there will be $400 000 in the account for William and still withdraw a payment? Explain your answer, including the regular payment amount to Charles. [3]

* Perpetuity ✓, payments = interest earned

$400\,000 \times 0.05 = \$20\,000$ ✓✓

∴ Pay himself $20 000 per year

(c) Charles hears of a friend Camilla who has a perpetuity paying interest at 7.5% compounded per annum that pays her $8000 quarterly. What is the present value that Charles would have to invest to receive this amount? [3]

$P \times \frac{0.075}{4} = 8000$ ✓

$\therefore P = \$426\,666.67$ ✓✓

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3. (17 marks)

Philip takes out a reducible interest loan of \$8000 which has interest compounded monthly on it and is to be repaid in monthly instalments of \$280.

Month	Amount owing at start of the month (\$)	Interest (\$)	Repayment (\$)	Amount owing at end of month (\$)
1	8000	96	280	7816
2	7816	93.79	280	***A***
3	***B***	***C***	280	***D***

(a) Show that the annual interest rate used is 14.4%. [2]

$\frac{96}{8000} \times 12 \times 100 = 14.4\%$ ✓✓ (2)

(b) Calculate the values of A, B, C and D. [4]

A = \$7629.79 ✓

B = \$7629.79 ✓

C = \$91.56 ✓

D = \$7441.35 ✓

(c) A recurrence relation $T_n = dT_{n-1} - e,\ T_0 = f$ can be used to express the loan repayment process shown in the table above where T_n is the amount owing at the end of month n.

(i) State the value of d. [1]

$\left(1 + \frac{0.144}{12}\right) = 1.012$ ✓

(ii) State the value of e. [1]

280 ✓

(iii) State the value of f. [1]

8000 ✓

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(d) How long will it take Philip to repay the loan under these circumstances? [1]

36 MONTHS = 3 yrs ✓

(e) How much interest will Philip pay altogether? [3]

PAID = 35 × 280 + 280 − 224.48 ✓

= $9855.52 ✓

∴ INTEREST = 9855.52 − 8000

= $1855.52 ✓

(f) Philip wants instead to make quarterly payments but still pay off the loan in the same length of time. How much would he have to increase his repayments by? [2]

N 12
I 14.4
PV 8000
PMT ?
FV 0
PY 4
CY 12

PMT = $834.87 ✓

∴ INCREASE PMT

= 834.87 − 280

= $554.87 ✓

(g) The bank refuses the quarterly payment plan but suggests they are happy to decrease Philip's original monthly repayment amounts to just $96. Should Philip take the new offer? Explain your answer. [2]

NO ✓

$96 is the interest accumulated on $8000 ∴ loan will never reduce. ✓

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4. (14 marks)

Harry is saving for his first car which will cost \$11 999. The bank offers him a one-year introductory offer on a savings account of 4.2% per annum compounded monthly. Harry initially invests \$7986 in the savings account. Interest is added at the end of each month just before Harry makes a regular monthly deposit of \$160.

(a) Express the saving process for the first year as a recursive formula. [2]

$T_{n+1} = 1.0035\,T_n + 160,\ T_0 = 7986$ ✓ ✓

(2)

(b) (i) How much has Harry saved after six months? [1]

\$9123.62 ✓

(1)

FT
$T_0 = 11\,999$
∴ \$13 221.63

(ii) How much of this amount is interest earned? [2]

$7986 + 6 \times 160 = 8946$ ✓

$\therefore$ INT $= 9123.62 - 8946$

$= \$177.62$

(2)

FT
$T_0 = 11\,999$
\$12 959 ✓
\$262.63 ✓

(c) Can Harry afford the car at the end of the first year? Justify your answer. [2]

NO ✓

only saved \$10 285.34 after 1 yr ✓

(2)

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If the account is still active after a year, the bank decreases its interest rate by 2.5% and continues to compound interest monthly.

(d) How many months altogether does it take for Harry to be able to afford the car? [2]

$\therefore$ NEW RATE $= 1.7\%$ ✓

$T_{n+1} = \left(1 + \frac{0.017}{12}\right) T_n + 160, \quad T_0 = 10285.34$

$\therefore$ 22 MONTHS ✓ (2)

(e) Harry decides to make cutbacks so that he can afford greater monthly deposits. What will he need to increase these payments by to save enough for the car in the first year? [2]

N 12
I 4.2
PV 7986
PMT ?
FV 11999
PY 12
CY 12

$\therefore$ PMT $= \$300.08$ ✓

INCREASE $= 300.08 - 160$

$= \$140.08$ ✓ (2)

FT if used $I = 2.5\%$
$\therefore \$160.51$ ✓

(c) When Harry finally buys the car, he finds it decreases in value to just \$5600 after three years. Assuming that the car depreciates at a rate of 46% for the first year and then at a constant rate in each of the following years, calculate the depreciation rate of the car in its third year. [3]

$11999 \times 0.54 = \$6479.46$ ✓

$\therefore 6479.46 \times x^2 = 5600$

$x = 0.93$ ✓

$\therefore$ Rate $= 7\%$ ✓ (3)

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5. (7 marks)

Anne and Liz are both investing $3800 into accounts compounding interest for 2 years.

(a) Anne places her money in an account earning interest at the rate of 2.280% per annum, compounded monthly.

(i) Complete the table below showing the value of Anne's investment at the end of the 2nd and third month. [2]

Number of months since money was invested	1	2	3	…	24
Value of investment ($)	3807.22	3814.45 ✓	3821.71 ✓	…	$3977.12

(ii) State the recursive rule for Anne's investment which gives the values shown in the table above [2]

$T_{n+1} = 1.0019 T_n$ ✓, $T_0 = 3800$ ✓

(b) Liz places her money in an account earning interest daily. After two years the value of both Anne's and Liz's investments are the same.

Explain how the change to the compounding period has affected the annual rate of interest required for the value of Liz's investment to be the same as that of Anne. Include calculations to support your answer. [3]

LIZ

N 365×2

I ?

PV −3800

PMT 0

FV 3977.12 ✓

PY 365 ✓

CY 365

∴ I = 2.278

∴ Slightly lower ✓ rate needed

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